## Swap (Hard - Level 1000)

Let's play with sequence of non negative integer. Given two sequence of $\mathbf{n}$ non negative integers $\left(a_{1}, a_{2} \ldots a_{n}\right)$ and $\left(b_{1}, b_{2} \ldots b_{n}\right)$. Both sequence has maximum element less than $\mathbf{k}$, max $\left(a_{1}, a_{2} \ldots\right.$ $\left.a_{n}\right)<\mathbf{k}$ and $\max \left(b_{1}, b_{2} \ldots b_{n}\right)<\boldsymbol{k}$. The game rule is you can edit both sequence with this operation: swap $a_{i}$ and $b_{i}$ with $1 \leq \boldsymbol{i} \leq \mathbf{n}$, and the goal is to make sequence $\mathbf{a}$ and $\mathbf{b}$ become increasing sequence: $a_{j} \leq a_{j}$ if and only if $i \leq j$ and $b_{i} \leq b_{j}$ if and only if $i \leq j$. But not all initial sequence $\mathbf{a}$ and $\mathbf{b}$ can be solved.

For example $(2,0)$ and $(0,1)$ is a pair of sequence that can't be solved:

- If you don't swap any element, you have $(2,0)$ and $(0,1)$, but sequence $(2,0)$ is not increasing.
- If you swap first element only, then the pair become like this $(0,0)$ and $(2,1)$, sequence $(2$, 1) is not increasing.
- If you swap second element only, then the pair become like this ( 2,1 ) and ( 0,0 ), again ( 2 , 1) is not increasing.
- If you swap both element, then the pair become like this $(0,1)$ and $(2,0)$, again $(2,0)$ is not increasing

So it's impossible to solve if initial sequence is $(2,0)$ and $(0,1)$, because all possible move can't make both sequence become increasing.

Now given $\mathbf{n}$ and $\mathbf{k}$, your task is to compute number of different pair of initial sequence $(\mathbf{a}, \mathbf{b})$ that can be solved with game described above.

## Input

First line there is an integer $\mathbf{T}$ denoting number of test case, then $\mathbf{T}$ test cases follow.
For each case, there are two integers $\mathbf{n}$ and $\mathbf{k}$ written in one line, separated by a space.

## Output

For each case, output number of different pair of initial sequence $(\mathbf{a}, \mathbf{b})$, since the answer can be large, output the answer modulo $10^{9}+7$.

## Constraints

$0<\mathbf{T} \leq 10^{4}$
$0<\min (\mathbf{n}, \mathbf{k}) \leq 1000$
$0<\max (\mathbf{n}, \mathbf{k})<10^{1000}$

## Example

## Input:

## Output:

1
4
9
11
26
46

## Explanation

Here is list of all possible pair of initial sequence $(\mathbf{a}, \mathbf{b})$ on each case:
Case 1: $\{[(0,0),(0,0)]\}$
Case 2: $\{[(0),(0)],[(0),(1)],[(1),(0)],[(1),(1)]\}$
Case 3: $\{[(0),(0)],[(0),(1)],[(0),(2)],[(1),(0)],[(1),(1)],[(1),(2)],[(2),(0)],[(2),(1)],[(2),(2)]\}$
Case 4: $\{[(0,0),(0,0)],[(0,0),(0,1)],[(0,0),(1,1)],[(0,1),(0,0)],[(0,1),(0,1)],[(0,1),(1,0)],[(0,1)$, $(1,1)],[(1,0),(0,1)],[(1,1),(0,0)],[(1,1),(0,1)],[(1,1),(1,1)]\}$

Case 5: $\{[(0,0,0),(0,0,0)],[(0,0,0),(0,0,1)],[(0,0,0),(0,1,1)],[(0,0,0),(1,1,1)],[(0,0,1),(0,0$, $0)],[(0,0,1),(0,0,1)],[(0,0,1),(0,1,0)],[(0,0,1),(0,1,1)],[(0,0,1),(1,1,0)],[(0,0,1),(1,1,1)]$, $[(0,1,0),(0,0,1)],[(0,1,0),(1,0,1)],[(0,1,1),(0,0,0)],[(0,1,1),(0,0,1)],[(0,1,1),(0,1,1)],[(0$, $1,1),(1,0,0)],[(0,1,1),(1,0,1)],[(0,1,1),(1,1,1)],[(1,0,0),(0,1,1)],[(1,0,1),(0,1,0)],[(1,0$, 1), (0, 1, 1)], [(1, 1, 0), (0, 0, 1)], [(1, 1, 1), (0, 0, 0)], [(1, 1, 1), (0, 0, 1)], [(1, 1, 1), (0, 1, 1)], [(1, 1, 1), $(1,1,1)]\}$

Case 6: $\{[(0,0),(0,0)],[(0,0),(0,1)],[(0,0),(0,2)],[(0,0),(1,1)],[(0,0),(1,2)],[(0,0),(2,2)],[(0,1)$, $(0,0)],[(0,1),(0,1)],[(0,1),(0,2)],[(0,1),(1,0)],[(0,1),(1,1)],[(0,1),(1,2)],[(0,1),(2,2)],[(0,2)$, $(0,0)],[(0,2),(0,1)],[(0,2),(0,2)],[(0,2),(1,0)],[(0,2),(1,1)],[(0,2),(1,2)],[(0,2),(2,0)],[(0,2)$, $(2,1)],[(0,2),(2,2)],[(1,0),(0,1)],[(1,0),(0,2)],[(1,1),(0,0)],[(1,1),(0,1)],[(1,1),(0,2)],[(1,1)$, $(1,1)],[(1,1),(1,2)],[(1,1),(2,2)],[(1,2),(0,0)],[(1,2),(0,1)],[(1,2),(0,2)],[(1,2),(1,1)],[(1,2)$, $(1,2)],[(1,2),(2,1)],[(1,2),(2,2)],[(2,0),(0,2)],[(2,1),(0,2)],[(2,1),(1,2)],[(2,2),(0,0)],[(2,2)$, $(0,1)],[(2,2),(0,2)],[(2,2),(1,1)],[(2,2),(1,2)],[(2,2),(2,2)]\}$

## Other Info

Test case ( $\mathbf{n}$ and $\mathbf{k}$ ) is generated randomly using this rule:

- Probability that $\mathbf{n}>\mathbf{k}$ or $\mathbf{n}<=\mathbf{k}$ is $\sim 50 \%$ each.
- Maximum $\mathbf{n}$ and $\mathbf{k}$ is random log-uniform.
- Minimum $\mathbf{n}$ and $\mathbf{k}$ is random uniform.

Explanation about my Algorithm complexity:
My 3.8KB of $C$ code with $O\left(\min (\mathbf{n}, \mathbf{k})^{\wedge} 3\right)$ complexity got $A C$ in 32.17 s .
Other submission like $O\left(\min (\mathbf{n}, \mathbf{k})^{\wedge} 4\right)$ in fast language, and $O\left(\min (\mathbf{n}, \mathbf{k})^{\wedge} 3\right)$ in slow language is all TLE. That's why this problem has "Hard" label.

Sorry for slow language user, I think it's impossible to solve this problem unless $\mathrm{O}\left(\min (\mathbf{n}, \mathbf{k})^{\wedge} 2\right)$ exists. I recommend to try Medium version first, or learn fast language :-P

About complexity, l've proved using math that no algorithm with complexity better than O (min(n, $\left.\mathbf{k})^{\wedge} 2\right)$, this is the lower bound. My best algorithm for now is $\mathrm{O}\left(\min (\mathbf{n}, \mathbf{k})^{\wedge} 3\right)$, this is the upper bound. So the optimal algorithm lies between that lower and upper bound. I still don't have proof that my algo is optimal, so there is possibility that there is an algorithm that better than $\mathrm{O}(\mathrm{min}(\mathbf{n}$, $\mathbf{k})^{\wedge} 3$ ).

Btw, if I found around $O\left(\min (\mathbf{n}, \mathbf{k})^{\wedge} 2\right)$ by myself, l'll set "Extreme" version (level 10000+) of this problem. But if there is someone who solve this problem in around $O\left(\min (\mathbf{n}, \mathbf{k})^{\wedge} 2\right)$, of course he/she has honor to set "Extreme" version of this problem.

Time limit $\sim 3 \times$ my program top speed.

See also: Another problem added by Tjandra Satria Gunawan

