

# Delta catheti II (Hard)

**(3, 4, 5)** is a famous Pythagorean triple, it gives a quick answer to the question:  
For a given integer  $d$ , is there a Pythagorean triple  $(a, b, c)$  such that  $b - a = d$ ?

A solution is  $(3d, 4d, 5d)$ , and in fact one can easily prove that the set of solutions is infinite, and that there is an obvious total order on those solutions.

Given  $n$ , you'll have to find the  $n$ th term of the sequence of solutions.

Geometrically, it is the study of right triangles for which the difference of the catheti are equal to  $d$ .

## Input

The first line of input contains an integer  $T$ , the number of test cases.

$2T$  lines follow. Each case is on two lines.

The first line of the case contains three integers  $n, d, m$ .

The second line contains an integer  $L$  and  $2L$  other integers  $(p, e)$ , which give the prime factorization of  $d$  in standard format ( $d = \text{product } p^e$ ).

## Output

For each test case, compute the  $n$ th term amongst the solutions  $(a, b, c)$  for the problem :  
 $a^2 + b^2 = c^2$  with  $b - a = d$  and  $0 < a < b < c$ .

As the answer could not fit in a 64-bit container, simply output your answer modulo  $m$ .

## Example

**Input:**

```
3
1 1 235813
0
3 21 1000
2 3 1 7 1
9 119 11
2 7 1 17 1
```

**Output:**

```
3 4 5
63 84 105
5 3 1
```

## Explanations

For the first case, the first solution is **(3, 4, 5)**, as  $4 - 3 = 1$ .

For the second case, the firsts solutions are:

**(15, 36, 39), (24, 45, 51), (63, 84, 105), (144, 165, 219), (195, 216, 291), (420, 441, 609), ...**

The third one is **(63, 84, 105)**.

For the third case, the first solutions are:

(24, 143, 145), (49, 168, 175), (57, 176, 185), (85, 204, 221), (136, 255, 289), (180, 299, 349),  
(196, 315, 371), (261, 380, 461), (357, 476, 595), (481, 600, 769), (616, 735, 959), ...

The 9th solution is (357, 476, 595), reduced modulo 11, we get (5, 3, 1).

## Constraints

$0 < T < 10^4$

$0 < n < 10^{18}$

$0 < d < 10^{14}$

$1 < m < 10^9$

$d$  is the product of two integers lower than  $10^7$ .

$n, d_1, d_2, m$  : Uniform randomly chosen in their range.

Those constraints are set to allow C-like users to work only with 64bit containers.

For your information, my 3kB-python3 code get AC in 2.80s with 12MB of memory print.

It should be much faster with a compiled language.

**Warning** : It's my hardest problem. Have fun ;-)